Sphere

1. The volume of a sphere is $\frac{4}{3}\pi r^3$ cubic units, then the ratio of the volume of a cube to that of a sphere which will fit inside the cube is

(A) $\frac{4}{3}:\pi$ (B) 6 : π (C) 4 : 3 (D) 4 : π

Sol. **(B)**

Volume of the sphere is $\frac{4}{3}\pi r^3$ cubic units next line side of the cube = diameter of the sphere which will fit inside the cube.

So, Side of the cube $= 2r \Rightarrow$ volume of the cube $= (2r)^3 = 8r^3$

$$\text{Required ratio} = \frac{8r^3}{\frac{4}{3}\pi r^3} = 6 : \pi.$$

2. A sphere of radius 9 cm is dropped into a cylindrical vessel of radius 10 cm. If the sphere is submerged completely, then its height (in cm) to which the water rises is :

(A) 2.35 (B) 2.30 (C) 9.72 (D) 2.15

Sol. **(C)**

Let the height to which the water rises be h

Then the volume of the sphere = Volume of the rising water

$$\frac{4}{3}\Pi(9)^3 = \pi(10)^2 h$$

$$\Rightarrow \quad h = 9.72 \text{ cm}$$

3. A hollow sphere of internal and external diameters 2 cm and 4 cm respectively is melted into a cone of base diameter 6 cm. Find the height of the cone :

(A) 28/9 cm (B) 12/5 cm (C) 16 cm (D) None of these

Sol. **(A)**

Volume of the hollow sphere= Volume of the formed cone of height h

$$\Rightarrow \quad \frac{1}{3}\Pi(3)^2 h = \frac{4}{3}\Pi(2^3 - 1^3)$$

$$\Rightarrow \quad h = 28/9 \text{ cm}.$$

4. The ratio of the volume and surface area of a sphere of unit radius.

(A) 4 : 3 (B) 3 : 4 (C) 1 : 3 (D) 3 : 1

Sol. **(C)**

Ratio of the volume & surface area of a sphere of unit radius

$$= \frac{\frac{4}{3}\pi(1)^3}{4\pi(1)^2} = 1 : 3.$$

5. The volumes of two spheres are in the ratio 125: 27. Find the difference of their surface areas, if the sum of their radii is 8 units.

(A) 64 π sq units (B) 78 sq units (C) 88 π sq units (D) 48 sq units

Sol. **(A)**

Ratio of volumes of two spheres is given as 125 : 27.

Then the ratio of their radii will be $(125)^{1/3} : (27)^{1/3}$ i.e. 5: 3.

Sum of their radii is given as 8 So their radii are 5 and 3 units respectively.

Difference of their surface areas $= 4\pi (5)^2 - 4\pi(3)^2$

$= 64\pi$ sq units.

6. The radius of a sphere is increased by P% Its surface area increases by :

(A) P% (B) P^2% (C) $\left(2P + \frac{P^2}{100}\right)\%$ (D) $\frac{P^2}{2}\%$

Sol. **(C)**

Let the radius of the sphere be *r* then

$$\text{It's surface area} = 4\pi r^2$$

$$\text{Increased radius} = r\left(1 + \frac{P}{100}\right)$$

$$\text{Increased surface area} = 4\pi r^2\left(1 + \frac{P}{100}\right)^2$$

$$\text{Increment in surface area} = 4\pi r^2\left(1 + \frac{2P}{100} + \frac{P^2}{(100)^2}\right) - 4\pi r^2$$

$$= 4\pi r^2\left(\frac{2P}{100} + \frac{P^2}{(100)^2}\right)$$

$$\% \text{ increase in Surface area} = \frac{4\pi r^2\left(\frac{2P}{100} + \frac{P^2}{(100)^2}\right)}{4\pi r^2} \times 100$$

$$= \left(2P + \frac{P^2}{100}\right)\%.$$

7. A hollow spherical bowl whose inner radius is 3 cm is full of water. Half of the water is transferred to a conical cup and it completely filled the cup. If the height of the cup is 9 cm, then the radius of the base of cone, in cm is.

(A) 4 (B) 8 π (C) $\sqrt{6}$ (D) 16

Sol. **(C)**

Volume of the water in hollow spherical ball $=\frac{4}{3}\Pi(3)^3=36\pi$

Half of the water in transferred to a conical cup and it completely filled the cup. Let the radius of the conical cup be r cm then. $\frac{1}{3}\Pi r^2(9)=36\pi/2$

$\Rightarrow$ $r = \sqrt{6}$ cm.

8. A sphere is melted to form a cylinder whose height is $15\frac{3}{16}$ times its radius. What is the ratio of radii of sphere to the cylinder ?

(A) 3 : 1 (B) 7 : 4 (C) 9 : 4 (D) Data inadequate

Sol. **(C)**

Volume of the sphere $= \frac{4}{3}\pi r^3$

and volume of the cylinder $= \pi(R)^2\left(15\frac{3}{16}\right)R,$ where r and R are radii of sphere and cylinder respectively.

Now $\frac{4}{3}\pi r^3 = \pi(R)^2\left(\frac{243}{16}\right)R$

$\Rightarrow$ $\frac{r^3}{R^3} = \frac{3^6}{2^6} \Rightarrow \frac{r}{R}=\frac{9}{4}=9:4.$

9. The curved surface areas of two spheres are in the ratio 9 : 25. Find the ratio of their volumes.

(A) 1 : 2 (B) 27/125 (C) 8 : 1 (D) 4 : 1

Sol. **(B)**

Let the radius of two spheres be r_1 and r_2 respectively then

$\frac{4\pi r_1^2}{4\pi r_2^2} = 9/25 \quad \Rightarrow \quad r_1 : r_2 = 3:5$

Now ratio of their volumes

$=\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \left(\left(3/5\right)^3=27/125\right.$

10. The curved surface areas of two spheres are in the ratio 1 : 9. Find the ratio of their volumes.
(A) 1 : 2 (B) 1 : 27 (C) 8 : 1 (D) 4 : 1

Sol. **(B)**

Let the radius of two spheres be r_1 and r_2 respectively then

$$\frac{4\pi r_1^2}{4\pi r_2^2} = 1/9 \qquad \Rightarrow \quad r_1 : r_2 = 1 : 3$$

Now ratio of their volumes

$$= \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = (1/3)3=1/27$$