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ಅರ್ಹತಾ / ಸ್ಪರ್ಧಾತ್ಮಕ ಪರೀಕ್ಷೆ - 2024

ಪತ್ರಿಕೆ-2

ಒಟ್ಟು ಪ್ರಶ್ನೆಗಳ ಸಂಖ್ಯೆ : 100

ಗರಿಷ್ಠ ಅಂಕಗಳು : 200

ನಿಮ್ಮ ನೋಂದಣಿ ಸಂಖ್ಯೆಯನ್ನು ನಮೂಂದಿಸಿ

A1

ವಿಷಯ : ಗಣಿತ ವಿಜ್ಞಾನಗಳು

ಅಭ್ಯರ್ಥಿಗಳಿಗೆ ಸೂಚನೆಗಳು

ಮಾಡಿ :

1. ಕೊಠಡಿ ಮೇಲ್ವಿಚಾರಕರು ಪ್ರಶ್ನೆಪತ್ರಿಕೆಯನ್ನು ನಿಮಗೆ ಬಿ. 9.55 ಕ್ಕೆ ಕೊಡುತ್ತಾರೆ.
2. ನೋಂದಣಿ ಸಂಖ್ಯೆಯನ್ನು ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯಲ್ಲಿ ಬರೆದು ಅದಕ್ಕೆ ಸಂಬಂಧಿಸಿದ ವೃತ್ತಗಳನ್ನು ಸಂಪೂರ್ಣವಾಗಿ ತುಂಬಿದ್ದೀರೆಂದು ಖಾತ್ರಿಪಡಿಸಿಕೊಳ್ಳತಕ್ಕದ್ದು.
3. ಪ್ರಶ್ನೆಪತ್ರಿಕೆಯ ವರ್ಷನ್ ಕೋಡ್ ಅನ್ನು ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯಲ್ಲಿ ಬರೆದು ಅದಕ್ಕೆ ಸಂಬಂಧಿಸಿದ ವೃತ್ತಗಳನ್ನು ಸಂಪೂರ್ಣವಾಗಿ ತುಂಬಬೇಕು.
4. ಪ್ರಶ್ನೆಪತ್ರಿಕೆಯ ವರ್ಷನ್ ಕೋಡ್ ಮತ್ತು ಕ್ರಮ ಸಂಖ್ಯೆಯನ್ನು ನಾಮಿನಲ್ ರೋಲ್‌ನಲ್ಲಿ ತಪ್ಪಿಲ್ಲದೆ ಬರೆಯಬೇಕು.
5. ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯ ಕೆಳಭಾಗದ ನಿಗದಿತ ಜಾಗದಲ್ಲಿ ಕಡ್ಡಾಯವಾಗಿ ಸಹಿ ಮಾಡಬೇಕು.

ಮಾಡಬೇಡಿ :

- ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯಲ್ಲಿ ಮುದ್ರಿತವಾಗಿರುವ ಟೈಮಿಂಗ್ & ಮಾರ್ಕ್‌ನ್ನು ತಿದ್ದಬಾರದು / ಹಾಳುಮಾಡಬಾರದು / ಅಳಿಸಬಾರದು.

ಅಭ್ಯರ್ಥಿಗಳಿಗೆ ಮುಖ್ಯ ಸೂಚನೆಗಳು

1. ಪ್ರಶ್ನೆಗಳಲ್ಲಿ ಬಳಸಿರುವ signs and symbols ಗಳನ್ನು, ಬೇರೆ ರೀತಿಯಲ್ಲಿ ಹೇಳಿದ ಹೊರತು, ನಿಗದಿತ ಪಠ್ಯಪುಸ್ತಕದಲ್ಲಿನ ಅರ್ಥವನ್ನು ಪರಿಗಣಿಸಬೇಕು.ಪೇಕು.
2. ಪ್ರಶ್ನೆಪತ್ರಿಕೆಯಲ್ಲಿ ಒಟ್ಟು 100 ಪ್ರಶ್ನೆಗಳಿದ್ದು, ಪ್ರತಿ ಪ್ರಶ್ನೆಗೂ 4 ಬಹು ಆಯ್ಕೆ ಉತ್ತರಗಳು ಇರುತ್ತವೆ. ಪ್ರತಿ ಪ್ರಶ್ನೆಯ ಕೆಳಗೆ ಕೊಟ್ಟಿರುವ ನಾಲ್ಕು ಬಹು ಆಯ್ಕೆಯ ಉತ್ತರಗಳಲ್ಲಿ ಸರಿಯಾದ ಒಂದು ಉತ್ತರವನ್ನು ಆಯ್ಕೆ ಮಾಡಿ.
3. ಬೆಳಿಗ್ಗೆ 10.00 ಗಂಟೆಗೆ ಪ್ರಶ್ನೆ ಪತ್ರಿಕೆಯ ಬಲಭಾಗದಲ್ಲಿರುವ ಸೀಲ್ ತೆಗೆದು ಈ ಪ್ರಶ್ನೆಪತ್ರಿಕೆಯಲ್ಲಿ ಯಾವುದೇ ಪುಟಗಳು ಮುದ್ರಿತವಾಗಿಲ್ಲದೇ ಇರುವುದು ಕಂಡು ಬಂದಲ್ಲಿ ಅಥವಾ ಹರಿದು ಹೋಗಿದ್ದಲ್ಲಿ ಅಥವಾ ಯಾವುದೇ ಅಕ್ಷರಗಳು ಬಿಟ್ಟುಹೋಗಿದ್ದಲ್ಲಿ ಪರೀಕ್ಷೆ ಪ್ರಾರಂಭವಾದ 5 ನಿಮಿಷಗಳೊಳಗೆ ಪ್ರಶ್ನೆಪತ್ರಿಕೆಯನ್ನು ಬದಲಾಯಿಸಿಕೊಳ್ಳುವುದು. ನಂತರ ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯಲ್ಲಿ ಉತ್ತರಿಸಲು ಪ್ರಾರಂಭಿಸುವುದು.
4. ಪ್ರಶ್ನೆಪತ್ರಿಕೆಯಲ್ಲಿನ ಪ್ರಶ್ನೆಗೆ ಅನುಗುಣವಾಗಿರುವ ಸರಿ ಉತ್ತರವನ್ನು ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯಲ್ಲಿ ಅದೇ ಕ್ರಮ ಸಂಖ್ಯೆಯ ಮುಂದೆ ನೀಡಿರುವ ಸಂಬಂಧಿಸಿದ ವೃತ್ತವನ್ನು ನೀಡಿ ಅಥವಾ ಕಪ್ಪು ಬಾಲ್ ಪಾಯಿಂಟ್ ಪೆನ್‌ನಿಂದ ಸಂಪೂರ್ಣ ತುಂಬುವುದು.

ಸರಿಯಾದ ಕ್ರಮ	ತಪ್ಪು ಕ್ರಮಗಳು WRONG METHODS											
CORRECT METHOD	❌	②	③	④	①	②	③	④	①	●	●	④
① ● ③ ④	①	②	③	④	①	●	③	④	①	②	③	④

5. ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯನ್ನು ಸ್ಕ್ಯಾನ್ ಮಾಡುವ ಸ್ಕ್ಯಾನರ್ ಬಹಳ ಸೂಕ್ಷ್ಮವಾಗಿದ್ದು ಸಣ್ಣ ಗುರುತನ್ನು ಸಹ ದಾಖಲಿಸುತ್ತದೆ. ಆದ್ದರಿಂದ ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯಲ್ಲಿ ಉತ್ತರಿಸುವಾಗ ಎಚ್ಚರಿಕೆ ವಹಿಸಿ.
6. ಪ್ರಶ್ನೆಪತ್ರಿಕೆಯಲ್ಲಿ ಕೊಟ್ಟಿರುವ ಖಾಲಿ ಜಾಗವನ್ನು ಕಚ್ಚಾ ಕೆಲಸಕ್ಕೆ ಉಪಯೋಗಿಸಿ. ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯನ್ನು ಇದಕ್ಕೆ ಉಪಯೋಗಿಸಬೇಡಿ.
7. ಮಧ್ಯಾಹ್ನ 1.00 ಗಂಟೆಗೆ ಕೊನೆಯ ಬೆಲ್‌ಕೋನ್ ಆದ ನಂತರ ಉತ್ತರಿಸುವುದನ್ನು ನಿಲ್ಲಿಸಿ, ಓ.ಎಂ.ಆರ್. ಉತ್ತರ ಪತ್ರಿಕೆಯನ್ನು ಕೊಠಡಿ ಮೇಲ್ವಿಚಾರಕರಿಗೆ ಯಥಾಸ್ಥಿತಿಯಲ್ಲಿ ನೀಡಿರಿ.
8. ಕೊಠಡಿ ಮೇಲ್ವಿಚಾರಕರು ಮೇಲ್ಕಾಗದ ಹಾಳೆಯನ್ನು ಪ್ರತ್ಯೇಕಿಸಿ (ಕಚೇರಿ ಪ್ರತಿ) ತಮ್ಮ ವಶದಲ್ಲಿ ಇಟ್ಟುಕೊಂಡು ಹಿಂಬದಿಯ ಯಥಾಪ್ರತಿಯನ್ನು (ಅಭ್ಯರ್ಥಿಯ ಪ್ರತಿ) ಅಭ್ಯರ್ಥಿಗಳಿಗೆ ಕೊಡುತ್ತಾರೆ.
9. ಎಲ್ಲಾ ಪ್ರಶ್ನೆಗಳಿಗೆ ಸಮಾನ ಅಂಕಗಳಿರುತ್ತವೆ.
10. ಯಾವುದೇ ರೀತಿಯ ಮೊಬೈಲ್ ಫೋನ್, ಕ್ಯಾಲ್ಕ್ಯುಲೇಟರ್ ಮತ್ತು ಇತರೆ ರೀತಿಯ ಎಲೆಕ್ಟ್ರಾನಿಕ್/ಕಂಪ್ಯೂಟರ್ ಸಾಧನಗಳು ಇತ್ಯಾದಿಗಳನ್ನು ಪರೀಕ್ಷಾ ಕೇಂದ್ರದ ಆವರಣದೊಳಗೆ ತರುವುದನ್ನು ನಿಷೇಧಿಸಿದೆ.

1. The eigenvectors of the matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ are :

(1) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -1 \end{bmatrix}$

(2) $\begin{bmatrix} i \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -i \end{bmatrix}$

(3) $\begin{bmatrix} 1 \\ i \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -i \end{bmatrix}$

(4) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

2. Which of the following matrix is positive definite ?

(1) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(2) $\begin{bmatrix} 1 & -2 \\ -2 & -5 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$

(4) $\begin{bmatrix} 1 & 0 \\ 0 & -4 \end{bmatrix}$

3. Suppose U and W are subspaces of V such that $\dim U = 4$, $\dim W = 5$, and $\dim V = 7$. Then $\dim(U \cap W)$ is :

(1) 4

(2) 5

(3) 7

(4) 1

4. Let T be a linear operator on $\mathbb{R}^2$ defined by $T(x, y) = (2x - 7y, 4x + 3y)$. Then the matrix representation of T relative to the basis $B = \{(1, 3), (2, 5)\}$ is :

(1) $\begin{bmatrix} 2 & -7 \\ 4 & 3 \end{bmatrix}$

(2) $\begin{bmatrix} 121 & 201 \\ -70 & -116 \end{bmatrix}$

(3) $\begin{bmatrix} 2 & 4 \\ -7 & 3 \end{bmatrix}$

(4) $\begin{bmatrix} 121 & 201 \\ 70 & 116 \end{bmatrix}$

5. For real symmetric matrices A and B , which of the following is true ?

(1) $AB = BA$

(2) AB is symmetric matrix

(3) All eigenvalues of AB are real if $AB = BA$

(4) AB is invertible if either A is invertible or B is invertible

Space for Rough Work

6. The set $\{ \tau \in \mathbb{C} : \sum_{n=1}^{\infty} \left(\frac{z-1}{z+1} \right)^n \text{ is absolutely convergent} \}$ is :

- (1) $\{ \tau \in \mathbb{C} : |\delta| < 1 \}$ (2) $\{ \tau \in \mathbb{C} : \operatorname{Re} z > 0 \}$
 (3) $\{ \tau \in \mathbb{C} : \operatorname{Im} z > 0 \}$ (4) $\mathbb{C}$

7. The radius of convergence of the power series $\sum_{n=1}^{\infty} \left(1 + \frac{2}{n^2} \right)^{n^2} z^n$ is :

- (1) e (2) $\frac{1}{e^2}$ (3) $\frac{1}{e}$ (4) 1

8. If a is in $\mathbb{C}$ and $r > 0$, then _____.

- (1) there is a number $\rho > 0$ such that $B_{\infty}(a; \rho) = B(a; r)$
 (2) there is a number $\rho > 0$ such that $B_{\infty}(a; \rho) \supset B(a; r)$
 (3) there is a number $\rho > 0$ such that $B_{\infty}(a; \rho) \supset \mathbb{C}$
 (4) there is a number $\rho > 0$ such that $B_{\infty}(a; \rho) \subset B(a; r)$

9. If we rotate the point $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2} \right)$, 2015 times about the origin through 60° in the clockwise direction, then the resulting position of that point in the plane is :

- (1) $(1, 0)$ (2) $(-1, 0)$
 (3) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2} \right)$ (4) $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2} \right)$

10. Let $f: U \rightarrow \mathbb{C}$ be an analytic function, where U is an open unit disk. Which of the following conditions do not imply f is constant on U ?

- (1) Real part of f is bounded on U
 (2) $|f|$ is constant on U
 (3) f assumes only imaginary values on U
 (4) f' is constant on U

Space for Rough Work

11. Let U be a connected subset of $\mathbb{C}$ and open in $\mathbb{C}$. Let $f: U \rightarrow \mathbb{C}$ be a non-constant analytic function. Let K be any compact subset of U . Then for any $\alpha \in \mathbb{C}$, the set $\{z \in K : f(z) = \alpha\}$ is :
- (1) uncountable (2) countable but not finite
(3) finite (4) finite but non-empty
12. Consider the $f: \mathbb{C} \rightarrow \mathbb{C}$ defined by $f(z) = z$. Then $|f|$ attains its minimum on the boundary B when B equals :
- (1) $B[0, 1]$ (2) $B[1, 2]$
(3) $B[i, \frac{1}{2}]$ (4) $B[i, 2]$
13. Let $U \subset \mathbb{C}$ be an open connected set and $f: U \rightarrow \mathbb{C}$ be analytic. Let $B = B[\alpha, R] \subset U$ and $M := \sup\{|f(z)| : z \in \partial B\}$. Then
- (1) $|f(z)| \leq M$ for all $z \in B$ (2) $|f(z)| \leq M$ for all $z \in U$
(3) $|f(z)| \geq M$ for all $z \in B$ (4) $|f(z)| \geq M$ for all $z \in U$
14. Let $\phi: \mathbb{C}_\infty \rightarrow \mathbb{C}_\infty$ be a bilinear map. Let $F = \{z \in \mathbb{C}_\infty : \phi(z) = z\}$. Then
- (1) F is non-empty (2) $|F|$ is infinite
(3) $|F| \leq 2$ (4) $|F| = 2$
15. Let $f(z) = z^4 e^{z^2}$ for $z \in \mathbb{C}$ and let $\gamma(t) = e^{it}$, $0 \leq t \leq 4\pi$. Then $\frac{1}{2\pi i} \int_{\gamma} \frac{f'(z)}{f(z)} dz$ equals :
- (1) 2 (2) 4
(3) 8 (4) 6
16. Which one of the following holds for all $n \in \mathbb{N}$?
- (1) $3^{n+1} < 2n + 2$ (2) $3^n < 2n + 1$
(3) $3^n > 2n + 1$ (4) $3^n \geq 2n + 1$

Space for Rough Work

17. Which of the following is a group ?

- (1) $\left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} \mid a \in \mathbb{Q}, a \neq 0 \right\}$ with respect to matrix multiplication
- (2) $\left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} \mid a \in \mathbb{Q}, a \neq 0 \right\}$ with respect to matrix addition
- (3) The set of all $n \times n$ matrices over $\mathbb{Q}$ with respect to matrix multiplication for $n \geq 1$
- (4) The set of all integers modulo n with respect to multiplication modulo n

18. The total number of non-trivial proper subgroups of a cyclic group of order 16 is :

- (1) 2 (2) 3 (3) 4 (4) 5

19. What is the largest order of an element in S_{10} , the symmetric group of degree 10 ?

- (1) 20 (2) 21
- (3) 22 (4) 24

20. The order of Sylow 2-subgroup of S_{2^n} , the symmetric group of degree 2^n is :

- (1) 2^n (2) 2^{2^n}
- (3) $2^{2^n - 1}$ (4) $2^{2^n + 1}$

21. Which of the following is a ring ?

- (1) The set $M_{n \times m}(\mathbb{R})$ of all $n \times m$ matrices over $\mathbb{R}$ with respect to matrix addition and matrix multiplication for $n \neq m$.
- (2) The set $GL_n(\mathbb{R})$ of all invertible $n \times n$ square matrices over $\mathbb{R}$ with respect to matrix addition and matrix multiplication.
- (3) The set $M_{n \times n}(\mathbb{R})$ of all $n \times n$ matrices over $\mathbb{R}$ with respect to matrix addition and matrix multiplication.
- (4) $2\mathbb{Z} \cup 3\mathbb{Z}$ with respect to usual addition and multiplication.

Space for Rough Work

22. Let $p(x)$ be an irreducible polynomial over a field F . Then which of the following statements is true ?
- (1) The ideal $\langle p(x) \rangle$ generated by $p(x)$ is a maximal ideal
 - (2) The quotient ring $\frac{F[x]}{\langle p(x) \rangle}$ has a non-trivial proper ideal
 - (3) No root of $p(x)$ lies in $\frac{F[x]}{\langle p(x) \rangle}$
 - (4) All roots of $p(x)$ are in $\frac{F[x]}{\langle p(x) \rangle}$
23. Let $F = \mathbb{Z}_2$. Then the splitting field of $x^3 + x^2 + 1 \in F[x]$ is a finite field with the number of elements equal to :
- (1) 8
 - (2) 10
 - (3) 12
 - (4) 18
24. Let α be a real cube root of 2 and let ω be a cube root of unity and $\omega \neq 1$. Then the degree of $Q(\alpha, \omega)$ over Q is :
- (1) 3
 - (2) 9
 - (3) 6
 - (4) 12
25. Let $\mathbb{F}_p = \mathbb{Z}_p$ and let $\mathbb{F}_{p^n}$ be a finite field with cardinality p^n and characteristic p . Then the cardinality of the Galois group $G(\mathbb{F}_{p^n}/\mathbb{F}_p)$ is :
- (1) n
 - (2) $\phi(n)$, the Euler ϕ -function
 - (3) $n - \phi(n)$
 - (4) $n + \phi(n)$
26. Pick out the topology on $\mathbb{R}$ so that the sequence $0, 1, 0, 1, 0, 1, \dots$ converge.
- (1) Cofinite topology
 - (2) Usual topology
 - (3) Discrete topology
 - (4) Ray topology (i.e. topology generated by the basis $\{(a, \infty) : a \in \mathbb{R}\}$)

Space for Rough Work

27. Consider $\mathbb{R}$ with co-countable topology τ_c . Find out which of the given function is continuous ?

- (1) $f : (\mathbb{R}, \tau_c) \rightarrow \mathbb{R}$ defined by $f(x) = x$
- (2) $f : (\mathbb{R}, \tau_c) \rightarrow \mathbb{R}$ defined by $f(x) = 2x^2 + 3$
- (3) $f : (\mathbb{R}, \tau_c) \rightarrow \mathbb{R}$ defined by $f(x) = 3$
- (4) $f : (\mathbb{R}, \tau_c) \rightarrow \mathbb{R}$ defined by $f(x) = \begin{cases} 0 & \text{if } x = 3 \\ -1 & \text{otherwise} \end{cases}$

28. With usual topology on $\mathbb{R}$ and the subspace equipped with subspace topologies, which of the following is **false** ? (The notation $X \simeq Y$ means the topological spaces X and Y are homomorphic)

- (1) $[a, b] \simeq [c, d]$
- (2) $(-1, 1) \simeq \mathbb{R}$
- (3) $[0, 1] \simeq (0, 1]$
- (4) $[-1, 1] \simeq \mathbb{R}$

29. In the Hilbert cube $[0, 1]^{\mathbb{N}}$, let τ_1 , τ_2 and τ_3 denote the following :

τ_1 : product topology

τ_2 : metric topology generated by the metric $d(x, y) = \sup_n \frac{|x_n - y_n|}{n}$

τ_3 : box topology

Pick out the true statement.

- (1) $\tau_1 \subseteq \tau_2$ but $\tau_2 \not\subseteq \tau_3$
- (2) $\tau_1 \subsetneq \tau_2$ and $\tau_2 \subseteq \tau_3$
- (3) $\tau_1 = \tau_2$ and $\tau_2 \subsetneq \tau_3$
- (4) $\tau_1 \subsetneq \tau_2 \subsetneq \tau_3$

30. Consider the subsets $S = (-1, 1) \times (-1, 1)$, $H = (1, \infty) \times \{0\}$ and $V = \{0\} \times (1, \infty)$ of $\mathbb{R}^2$ with the subspace topology of $\mathbb{R}^2$ with usual Euclidean metric d_2 . Which of the following sets is connected ?

- (1) $S \cup \{(1, 0)\} \cup V$
- (2) $S \cup \{(0, 1)\} \cup H$
- (3) $S \cup \{(1, 0)\} \cup H$
- (4) $S \cup V \cup H$

Space for Rough Work

31. A particular integral of $y'' + \omega^2 y = \sin(\omega x) \tan(\omega x)$, where ω is constant, is given by :

(1) $\frac{1}{2\omega^2} [\omega x \cos(\omega x) + 2 \sin(\omega x) \log \cos(\omega x)]$

(2) $\frac{1}{\omega^2} [\omega x \cos(\omega x) + 2 \cos(\omega x) \log \cos(\omega x)]$

(3) $\frac{1}{2\omega^2} [\omega x \sin(\omega x) + 2 \cos(\omega x) \log \cos(\omega x)]$

(4) $\frac{1}{\omega^2} [\omega x \sin(\omega x) - 2 \sin(\omega x) \log \sin(\omega x)]$

32. The functions $\exp(r_1 x)$, $\exp(r_2 x)$ and $\exp(r_3 x)$, where r_1 , r_2 and r_3 are distinct, are :

(1) Linearly dependent

(2) Only mutually linearly dependent

(3) Only mutually linearly independent

(4) Linearly independent

33. The complete solution of the differential equation

$$(x+2)^2 y'' - (x+2) y' + y = 0, \text{ is :}$$

(1) $A(x+2) + Bx(x+2)$

(2) $A(x+2) + B(x+2) \log(x+2)$

(3) $A \sin x + B \cos x \log(x+2)$

(4) $A(x+2) + B(x+2) \log|x+2|$

34. The solution of $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} 4 & -3 \\ 6 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ is given by :

(1) $A \begin{bmatrix} 3 \\ -2 \end{bmatrix} \exp(2t) + B \begin{bmatrix} 3 \\ 1 \end{bmatrix} \exp(-5t)$

(2) $A \begin{bmatrix} 3 \\ 2 \end{bmatrix} \exp(2t) + B \begin{bmatrix} 1 \\ 3 \end{bmatrix} \exp(-5t)$

(3) $A \begin{bmatrix} 3 \\ 4 \end{bmatrix} \exp(3t) + B \begin{bmatrix} 3 \\ 1 \end{bmatrix} \exp(-5t)$

(4) $A \begin{bmatrix} 1 \\ 2 \end{bmatrix} \exp(2t) + B \begin{bmatrix} 3 \\ 2 \end{bmatrix} \exp(-5t)$

Space for Rough Work

35. Let $u(x, y)$ be the solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 64$ in the unit disc $\{(x, y) = x^2 + y^2 < 1\}$ and

such that u vanishes on the boundary of the disc. Then $u\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ is equal to :

(1) 7

(2) -7

(3) 16

(4) -16

36. The solution of $u(x+y)u_x + u(x-y)u_y = x^2 + y^2$ with the Cauchy data $u = 0$ on $y = 2x$ is :

(1) $7u = 6xy + 4(x^2 + y^2)$

(2) $7u^2 = 3xy + 2(x^2 - y^2)$

(3) $7u^2 = 6xy + 4(x^2 - y^2)$

(4) $7u = 6xy + 4(x^2 - y^2)$

37. For the equation $L(y) = a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$, the adjoint operator L^* is given by $L^*(z) = (za_0)'' - (za_1)' + za_2$. Then the Lagrange identity for L is :

(1) $zL(y) - yL^*(z)$

(2) $yL(y) - zL^*(z)$

(3) $zL(y) + yL^*(z)$

(4) $z^2L(y) - y^2L^*(z)$

38. The integral of the Pfaffian $yz dx + 2xz dy - 3xy dz = 0$ is :

(1) $u(x, y, z) = \frac{x^2y^2}{z} = c$

(2) $u(x, y, z) = \frac{xy^2}{z^3} = c$

(3) $u(x, y, z) = \frac{xy}{z^3} = c$

(4) $u(x, y, z) = \frac{xy}{z^2} = c$

Space for Rough Work

39. The initial displacement f and initial velocity g appearing in D'Alembert's solution satisfy
- (1) $f, g \in c^2$ (2) $f, g \in c^1$
 (3) $f \in c^2$ and $g \in c^1$ (4) $f, g \in c^\infty$
40. The canonical form of $u_{xx} + x^2 u_{yy} = 0$ is given by :
- (1) $u_{\alpha\alpha} - u_{\beta\beta} = 0$ (2) $u_{\alpha\alpha} + u_{\beta\beta} = u_\alpha$
 (3) $u_{\alpha\alpha} + u_{\beta\beta} = -\frac{1}{2\alpha} u_\alpha$ (4) $u_{\alpha\alpha} + u_{\beta\beta} = \frac{1}{3\beta} u_\beta$
41. The Monge cone for $p^2 + q^2 = 1$ at $(0, 0, 0)$ is given by :
- (1) $x^2 - y^2 = z^2$ (2) $x^2 + y^2 = z^2$
 (3) $x^2 - y^2 = 2z^2$ (4) $x^2 + y^2 = (z + 1)^2$
42. The general integral of $xpq + yq^2 = 1$ is given by :
- (1) $(2z + b)^2 = 4ax + y$
 (2) $(2z + b)^2 = 4(ax + y)$
 (3) $(z + b)^2 = 4(ax + y)$
 (4) $(z + b)^3 = 2(ax + y)$
43. Find the third Picard approximate y_3 of $y' = y$, $y(0) = 2$ at 0.8, correct to two decimal places accuracy and its deviation d from the exact solution.
- (1) $y_3 = 4.24$, $d = 0.01$ (2) $y_3 = 4.24$, $d = 0.21$
 (3) $y_3 = 4.41$, $d = 0.04$ (4) $y_3 = 4.41$, $d = 0.09$
44. The step size h that can be used in the tabulation of $f(x) = \sin x$ in the interval $[1, 3]$ so that the linear interpolation will be correct to four decimal places after rounding is :
- (1) $h = 0.03$ (2) $h \leq 0.02$
 (3) $h \leq 0.03$ (4) $h \leq 0.04$

Space for Rough Work

45. The three term Taylor series solution for $v''' + vv'' = 0$ satisfying $v(0) = 0$, $v'(0) = 0$ and $v''(0) = 1$ and the bound on the error committed for $t \in [0, 0.2]$ are respectively given by

- (1) $\frac{t^2}{2!} - \frac{t^5}{5!} + \frac{t^8}{8!}$ and $(0.12) 10^{-10}$
- (2) $\frac{t^2}{2!} - \frac{t^5}{5!} + \frac{11}{8!} t^8$ and $(1.06) 10^{-11}$
- (3) $\frac{t^2}{2!} + \frac{3t^3}{3!} - \frac{5t^7}{7!}$ and $(0.13) 10^{-11}$
- (4) $\frac{3}{2!} t^2 - \frac{5}{3!} t^3 + \frac{11}{8!} t^8$ and $(1.11) 10^{-10}$

46. For what values of the step size h , the Euler's method applied to $y' = \alpha y$ ($\alpha < 0$), $y(0) = 1$ is stable?

- (1) $-1 < \alpha h < 0$
- (2) $-1 < \frac{\alpha h}{2} < 0$
- (3) $-3 < \alpha h < -2$
- (4) $-2 < \alpha h < 0$

47. The extremal of $\int_0^{\pi/2} (y'^2 + z'^2 + 2yz) dx$, $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 1$, $z(0) = 0$ and $z\left(\frac{\pi}{2}\right) = -1$ is :

- (1) $y = \cos x$, $z = -\cos x$
- (2) $y = e^{-x}$, $z = \cos x$
- (3) not existing
- (4) $y = \sin x$, $z = -\sin x$

Space for Rough Work

48. The solution of the Abel integral equation $f(x) = \int_0^x \frac{y(t)}{(x-t)^\alpha} dt$, $0 < \alpha < 1$ is given by :

(1) $\frac{\sin \alpha \pi}{\pi} \frac{d}{du} \left[\int_0^u \frac{f(x)}{(x-u)^{1-\alpha}} dx \right]$

(2) $\frac{\sin \alpha \pi}{\pi} \frac{d}{dt} \left[\int_0^t \frac{f(x)}{(t-x)^{1-\alpha}} dx \right]$

(3) $\frac{\cos \alpha \pi}{\pi} \frac{d}{dt} \left[\int_0^t \frac{f(t)}{(t-x)^{1-\alpha}} dx \right]$

(4) $-\frac{\sin \alpha \pi}{\pi} \frac{d}{dt} \left[\int_0^t \frac{f(x-t)}{(t-x)^{1-\alpha}} dx \right]$

49. The n^{th} iterated kernel $K_3(x, t)$ of $y(x) = \frac{5x}{6} + \frac{1}{2} \int_0^1 xt y(t) dt$ is :

(1) $\frac{1}{6} xt$

(2) $\frac{1}{9} xt$

(3) $\frac{1}{27} x^2 t$

(4) $-\frac{1}{9} xt$

50. The Hamilton – Jacobi equation for the problem of Harmonic oscillator (simple mass-spring system), in the usual notations, is given by :

(1) $\frac{\partial s}{\partial t} + \frac{p_x^2}{m} + kx^2 = 0$

(2) $-\frac{\partial s}{\partial t} + \frac{p_x^2}{m} + kx^2 = 0$

(3) $\frac{\partial s}{\partial t} + \frac{p_x^2}{2m} + \frac{1}{2} kx^2 = 0$

(4) $\frac{\partial s}{\partial t} - \frac{p_x^2}{2m} + \frac{1}{2} kx^2 = 0$

Space for Rough Work

51. Atwood's machine is an example for a :
- (1) Conservative system with holonomic, rheonomic constraint.
 - (2) Conservative system with holonomic, unilateral constraint.
 - (3) Conservative system with holonomic, scleronomic constraint.
 - (4) Conservative system with non-holonomic, unilateral constraint.
52. A spring of force constant 3 Nm^{-1} is connected between two identical simple pendulums, each of length 0.8 m . Calculate the period of the other pendulum if one is damped taking the mass of each bob as 0.23 kg ($g = 10 \text{ m/s}^2$).
- (1) 1.02 s
 - (2) 1.24 s
 - (3) 1.72 s
 - (4) 2.02 s
53. Consider a hoop rolling, without slipping, down an inclined plane. Then the constraint is :
- (1) Rheonomic involving first order derivatives
 - (2) Rheonomic involving second order derivatives
 - (3) Holonomic involving first order derivatives
 - (4) Non-holonomic involving first order derivatives
54. Which of the following is **not** the generating function of the four basic canonical transformations ?
- (1) $F = F_1(q, Q, t)$
 - (2) $F = F_2(q, P, t) - Q_i P_i$
 - (3) $F = F_3(p, Q, t) + q_i p_i$
 - (4) $F = F_4(p, P, t) - q_i P_i - Q_i P_i$
55. Consider the linear programming problem :
- $\max z = 3x + 4y$
- s. c. $5x + 4y \leq 200$
- $8x + 4y \leq 80$
- $5x + 4y \leq 100$
- What is the optimum solution ?
- (1) 60
 - (2) 40
 - (3) 100
 - (4) None of the above

Space for Rough Work

56. The point of intersection of Less than Ogive and More than Ogive corresponds to :

- (1) Mean
- (2) Median
- (3) Mode
- (4) Geometric mean

57. If $\bar{X} = 10$ and $Y = 2X - 20$, then the mean of Y values will be :

- (1) 20
- (2) 40
- (3) 0
- (4) 2

58. The joint probability density function of two random variables X and Y is given by

$$f(x, y) = x + y \text{ for } 0 \leq x \leq y, y \leq 1$$
$$= 0 \text{ otherwise.}$$

Then the marginal density of X is :

- (1) $f(x) = x + 1$
- (2) $f(x) = 1 + x + y$
- (3) $f(x) = x + \frac{1}{2} - \frac{3x^2}{2}$
- (4) $f(x) = x + \frac{1}{x}$

59. The condition for $\frac{X_n}{Y_n} \xrightarrow{P} \frac{X}{Y}$ is :

- (1) $P(Y_n = 0) = 0 \quad \forall n$
- (2) $P(Y_n = 0) = 0$ and $P(Y = 0) = 0$
- (3) $P(Y = 0) = 0$
- (4) $P(X_n = 0) \text{ and } P(X = 0) = 0$

Space for Rough Work

60. If the transition probability of a Markov chain is

$$\begin{pmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix},$$

then the steady-state distribution of the chain is :

(1) $\left(\frac{1}{3}, \frac{2}{3}, \frac{1}{2}\right)$

(2) $\left(\frac{1}{3}, \frac{10}{27}, \frac{8}{27}\right)$

(3) $\left(\frac{8}{27}, \frac{9}{27}, \frac{10}{27}\right)$

(4) $\left(\frac{1}{2}, \frac{1}{3}, \frac{2}{3}\right)$

61. A persistent state of a Markov chain is said to be null persistent if its mean recurrence time is :

(1) Infinite

(2) Finite

(3) One

(4) Zero

62. Steel rods are manufactured to be 3 inches in diameter but they are acceptable if they are between 2.99 inches and 3.01 inches. 5% are rejected as oversize and 5% are rejected as undersize. Assuming the diameter is normally distributed, the standard deviation of the distribution is :

(1) $\frac{1}{172}$

(2) $\frac{1}{286}$

(3) $\frac{1}{165}$

(4) $\frac{1}{20}$

Space for Rough Work

63. The ratio of two chi-square variates X_1 and X_2 with degrees of freedom n_1 and n_2 is distributed as :

- (1) Gamma Distribution (2) F-Distribution
(3) Beta Distribution (4) Normal Distribution

64. Let X_1, X_2 be independent identically distributed Poisson (θ) variates. Which of the following statistic is sufficient for θ ?

- (1) $X_1 + 2X_2$ (2) $X_1 - 3X_2$
(3) $X_1 + 3X_2$ (4) $X_1 + X_2$

65. Find the lower bound of the variance for unbiased estimators of the parameter θ for the density, $f(x, \theta) = \frac{1}{\pi[1+(x-\theta)^2]}$, $-\infty < x < \infty$.

- (1) $\frac{1}{n+1}$ (2) $\frac{2}{n}$ (3) $\frac{\theta}{n}$ (4) $\frac{n}{\theta}$

66. Find a best critical region of size $\alpha = 0.2$ for testing $H_0 : \theta = 0$ against $H_1 : \theta \neq 0$, based on a single value of x , for given $f(x, \theta) = 1 + \theta^2(x - \frac{1}{2})$, $0 \leq x \leq 1, 0 \leq \theta \leq \sqrt{2}$.

- (1) $\{x : x \leq 0.9\}$ (2) $\{x : x \leq 0.8\}$
(3) $\{x : x \geq 0.8\}$ (4) $\{x : x \geq 0.9\}$

67. Let $f(x, \theta) = \frac{2(\theta-x)}{\theta^2}$, $0 < x < \theta$, find the Likelihood Ratio Test statistic $\lambda(x)$, of $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ based on a sample of size 1.

- (1) $\lambda(x) = \frac{\theta_0 - x}{\theta_0^2}$ (2) $\lambda(x) = \frac{2(\theta_0 - x)}{\theta_0^2}$
(3) $\lambda(x) = \frac{2x(\theta_0 - x)}{\theta_0^2}$ (4) $\lambda(x) = \frac{4x(\theta_0 - x)}{\theta_0^2}$

Space for Rough Work

68. The median test for testing the equality of location parameters of two population may require the probability distribution, namely :

- | | |
|------------------------------|----------------------------|
| (1) Exponential distribution | (2) Geometric distribution |
| (3) Chi-square distribution | (4) Poisson distribution |

69. The Kolmogorov – Smirnov test is based on :

- | | |
|-------------------------------|---------------------------------|
| (1) Lehmann – Scheffé theorem | (2) Glivenko – Cantelli theorem |
| (3) Rao – Blackwell theorem | (4) Uniqueness theorem |

70. The covariance of the least square estimator in a full-rank model $Y = X\beta + e$, is given by :

- | | |
|--------------------------|----------------------|
| (1) σ^2 | (2) $\sigma^2(X'X)$ |
| (3) $\sigma^2(X'X)^{-1}$ | (4) $\sigma^2/n - p$ |

71. Which assumption in regression is violated when the response variable Y is categorical ?

- | | |
|------------------|-----------------------|
| (1) Independency | (2) Multicollinearity |
| (3) Normality | (4) Additivity |

72. Which of the following matrix form a quadratic form and possess the property of indefinite ?

- | | |
|--|--|
| (1) $\begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$ | (2) $\begin{pmatrix} 1 & -1 \\ 1 & -2 \end{pmatrix}$ |
| (3) $\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$ | (4) $\begin{pmatrix} 8 & -4 \\ -4 & 2 \end{pmatrix}$ |

73. If $X \sim N_p(\mu, \Sigma)$ and $Y = AX + B$, where A is $q \times p$ matrix, c is any $q \times 1$ vector, then Y follows :

- | | |
|---|---|
| (1) $(p - 1)$ – variate normal distribution | (2) $(q - 1)$ – variate normal distribution |
| (3) q – variate normal distribution | (4) p – variate normal distribution |

Space for Rough Work

74. Which sampling procedure is used if a sample of units was selected from each cluster, when a population was divided into clusters and it was found that within clusters, variation was less than the variation between clusters ?
- (1) Stratified sampling (2) Systematic sampling
(3) Cluster sampling (4) Multistage sampling
75. In the K-means algorithm for partitioning, each cluster is represented by the _____ of objects in the clustering.
- (1) Mode (2) Means
(3) Median (4) Members
76. In the process of confounding the block effect is mixed with :
- (1) Error effect (2) All treatment effects
(3) Unimportant treatment effect (4) Only important treatment effects
77. When the response variable is disturbed by another variable in Design of Experiments, a suitable method is :
- (1) Randomized Block Design (2) BIBD
(3) PBIBD (4) Analysis of Covariance
78. Which distribution is suitable for describing infant mortality data of the population ?
- (1) Uniform distribution (2) Weibull distribution
(3) Exponential distribution (4) Normal distribution
79. Which of the following statements is true in the context of Linear Programming Problem ?
- (1) A collection of all basic solutions to a Linear Programming Problem constitutes a convex set.
(2) There are infinite number of basic feasible solutions within feasible solution space.
(3) Every extreme point of the convex set of all feasible solutions is a basic feasible solution.
(4) Optimal solution does not exist if the feasible solutions form a convex polyhedron.
80. A company uses 5 lakh units of an item annually, each costing ₹ 1.60, each order costs ₹ 50/- and inventory carrying costs 20% of annual average inventory value. Find the economic order quantity.
- (1) 1250 units (2) 12500 units
(3) 125000 units (4) 14400 units

Space for Rough Work

81. Consider the sequence $u_n = \left(1 + (-1)^n \frac{1}{n}\right)^n$. Then :

- (1) $\lim_{n \rightarrow \infty} \sup u_n = \lim_{n \rightarrow \infty} \inf u_n = 1$ (2) $\lim_{n \rightarrow \infty} \sup u_n = \lim_{n \rightarrow \infty} \inf u_n = \frac{1}{e}$
 (3) $\lim_{n \rightarrow \infty} \sup u_n = \lim_{n \rightarrow \infty} \inf u_n = e$ (4) $\lim_{n \rightarrow \infty} \sup u_n = e, \lim_{n \rightarrow \infty} \inf u_n = \frac{1}{e}$

82. Which of the following statements is true ?

- (1) The set of all rational numbers is uncountable.
 (2) A complete metric space without isolated points is uncountable.
 (3) If A and B are countable, then the cartesian product $A \times B$ is uncountable.
 (4) The set of all irrational numbers is countable.

83. Which of the following series is **not** convergent ?

- (1) $\sum_{n=1}^{\infty} \frac{1}{2n^2 + n}$ (2) $\sum_{n=1}^{\infty} \frac{3n^3 - 2n^2 + 4}{n^7 - n^3 + 2}$
 (3) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ (4) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

84. The number of surjective maps from a set of 5 elements to a set of 3 elements is :

- (1) 150 (2) 243 (3) 125 (4) 124

85. $\lim_{n \rightarrow \infty} \left[\frac{1}{n^2} + \frac{2}{n^2} + \dots + \frac{n}{n^2} \right]$ is :

- (1) $\frac{1}{2}$ (2) 0 (3) 1 (4) ∞

86. The continuous image of a connected metric space is :

- (1) connected (2) not connected
 (3) compact (4) not compact

Space for Rough Work

87. Let $f: [0, 2] \rightarrow \mathbb{R}$ be defined by:

$$f(x) = \begin{cases} 1 & \text{for } x = 1 \\ 0 & \text{for } x \neq 1 \end{cases}$$

and $g: [0, 1] \rightarrow \mathbb{R}$ be defined by:

$$g(x) = \begin{cases} 1 & \text{for } x \text{ irrational} \\ 0 & \text{for } x \text{ rational} \end{cases}$$

Then

- (1) f and g are Riemann integrable
- (2) neither f nor g are Riemann integrable
- (3) f is Riemann integrable but g is not Riemann integrable
- (4) g is Riemann integrable but f is not Riemann integrable

88. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function such that $f(0) = 1$. If the derivative of f is less than 0 for all $x \in \mathbb{R}$, then f is:

- (1) bounded below on $\mathbb{R}$
- (2) bounded above on $[0, \infty)$
- (3) bounded above on $(-\infty, 0]$
- (4) bounded on $(\infty, 0)$

89. If $\sum f_n(x)$ converges uniformly to f on a set S , then

- (1) $f_n \rightarrow f$ pointwise
- (2) $f_n \rightarrow 0$ uniformly on S
- (3) $f_n \rightarrow 0$ on S but not uniformly
- (4) $f_n - f \rightarrow 0$ uniformly on S

90. Let f be differentiable function in (a, b) . Which of the following statements is **not** correct?

- (1) If $f'(x) \geq 0$ for all $x \in (a, b)$, then f is monotonically increasing.
- (2) If $f'(x) \leq 0$ for all $x \in (a, b)$, then f is monotonically increasing.
- (3) If $f'(x) \leq 0$ for all $x \in (a, b)$, then f is monotonically decreasing.
- (4) If $f'(x) = 0$ for all $x \in (a, b)$, then f is a constant function.

Space for Rough Work

91. Which of the following is **not** uniformly continuous on $(0, 1)$?

- (1) e^x (2) $\cos x$ (3) $\sin x$ (4) $\frac{1}{x}$

92. Which of the following statements is **false**?

- (1) If f is monotonic on $[a, b]$, then f is Riemann integrable.
 (2) If f is continuous on $[a, b]$, then f is Riemann integrable.
 (3) If the set of points of discontinuity of a function f on $[a, b]$ is finite then f is Riemann integrable.
 (4) If the set of points of discontinuity of a function f on $[a, b]$ is infinite then f is Riemann integrable.

93. Pick out those functions of $F : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ for which the derivative given by the Jacobian J_F will be :

$$\begin{pmatrix} 2x_1x_2 & x_1^2 & 0 & 0 \\ x_2x_3 & x_1x_3 & x_1x_2 & 0 \\ 2x_1 & 2x_2 & x_4^2 & 2x_3x_4 \end{pmatrix} x_2^2$$

- (1) $F(x_1, x_2, x_3, x_4) = (x_2^2 x_1^2, x_1 x_2 x_3, x_1^2 + x_3 x_4^2)$
 (2) $F(x_1, x_2, x_3, x_4) = (x_1^2 x_2, x_1 x_2 x_3, x_1^2 + x_2^2 + x_3 x_4^2)$
 (3) $F(x_1, x_2, x_3, x_4) = (x_1^3 x_2, x_1 x_2 x_3, 2x_3^2 x_4)$
 (4) $F(x_1, x_2, x_3, x_4) = (x_1 x_2^2, 2x_1 x_2 x_3, x_3 x_4^2)$

94. Which of the following is a subspace of vector space $\mathbb{R}^3$ over $\mathbb{R}$?

- (1) $W = \{(x, y, z) \in \mathbb{R}^3 \mid y = x^2\}$
 (2) $W = \{(x, y, z) \in \mathbb{R}^3 \mid x - 4y + 5z = 2\}$
 (3) $W = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 \leq 1\}$
 (4) $W = \{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0\}$

Space for Rough Work

95. The row space of a 20×40 matrix A has dimension 14. What is the dimension of the space of solutions of $Ax = 0$?

- (1) 26 (2) 14 (3) 34 (4) 6

96. If A is a 2×2 matrix over $\mathbb{R}$ with $\det(A + I) = 1 + \det A$, then

- (1) $\text{Tr}(A) = 0$ (2) $\det(A) = 0$
(3) $A = 0$ (4) $\det(A) \neq 0$

97. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ satisfies the matrix equation $A^2 - kA + 2I = 0$, then what is the value of k ?

- (1) 0 (2) 1 (3) 2 (4) 3

98. The dimension of the vector space of all symmetric matrices $A = (a_{ij})$ of order n ($n \geq 2$) with real entries, $a_{11} = 0$ and trace zero is :

- (1) $\frac{n^2 + n - 4}{2}$ (2) $\frac{n^2 - n + 4}{2}$
(3) $\frac{n^2 + n - 3}{2}$ (4) $\frac{n^2 - n + 3}{2}$

99. Let $M_3(\mathbb{R})$ be the set of all 3×3 real matrices. If $A \in M_3(\mathbb{R})$ has $1, -1, \frac{1}{3}$ as eigenvalues, then the eigenvalues of A^{-5} are :

- (1) $1, -1, 243$ (2) $1, 1, \frac{1}{243}$
(3) $1, -1, 3$ (4) $1, -1, -\frac{1}{243}$

100. Which of the following matrix is **not** diagonalizable over $\mathbb{R}$?

- (1) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
(3) $\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ (4) $\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$

Space for Rough Work